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SEISMIC PLASTIC DEFORMATION IN THE FREE FIELD

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SUMMARY

An analytical solution for the free-field seismic elastoplastic deformation of granular dry soil is developed. This solution is possible since the stress field solution from pseudostatic analysis is known at any acceleration level. A plastic potential function with two plastic material parameters is suggested to model the soil. This model can describe both the volume dilation and contraction of soil under shearing. Closed-form deformation solutions are derived for two special cases and the results from a numerical evaluation of the solution are compared with those measured from shaking table tests. From the deformation solution the shape of the horizontal plastic displacement distribution with depth is identical to that for the elastic deformation since both depend only on the distribution of the elastic shear modulus. Copyright © 1999 John Wiley & Sons, Ltd.

Key words: seismic free field; pseudostatic; granular soil; plastic deformation

INTRODUCTION

Seismic response of soil in the free field is the most basic problem in seismic soil mechanics. The seismic loading in the free field is usually idealized as a one-dimensional shear wave propagating vertically from bedrock to the ground surface. The free field problem has been solved using the wave equation analytically,[1–6] by numerical solution[7] and by the finite element method.[8] However in all these methods, an important property of soil, namely its plasticity, is not considered.

Under moderate seismic loading, the soil may yield therefore no longer respond elastically. Based on pseudostatic analysis, Richards *et al.*[9] derived a seismic full-range elastic and post-yield plastic stress solution for soil in the free field where the soil is idealized as an elastic perfectly-plastic material with a Mohr–Coulomb yield criterion. In this stress solution, not only does the shear stress vary linearly with the acceleration, but also the horizontal stress changes non-linearly due to progressive yield.

The plastic deformation of the soil can be very important since it is necessary to predict the movement of the ground surface, assess the potential of liquefaction, and to analyse soil–structure interaction. A complete free field solution should include both stress and deformation fields. Currently there is no analytical method to determine soil deformation in the free field beyond first

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yield. This paper provides such a solution for the plastic deformation of dry granular soil in the free field due to the earthquake loading.

For a shallow layer of soil near the ground surface, such as with the dimensions of retaining walls and other soil structures, the transient wave propagating effect is not of real importance. Therefore in this paper the free field seismic response of soil is analysed by the pseudostatic analysis method with a constant peak value of horizontal accelerations. The results are conservative under such assumption.[10]

FREE-FIELD STRESS SOLUTION

Consider a homogeneous horizontal layer of granular soil of infinite lateral extent with unit weight γ. For simplicity, let us neglect vertical acceleration although this effect can be included in a straightforward way.[9] If any distribution of horizontal acceleration $k_h g$ is applied to the soil layer as in Figure 1, the one dimensional stress field can be expressed as

$$\sigma_x = K\gamma z; \qquad \sigma_z = \gamma z; \qquad \tau_{xz} = -\, k_h \gamma z \tag{1}$$

where K is the coefficient of lateral earth pressure.

At rest or in the elastic range, the coefficient of lateral earth pressure K is often taken as

$$K = K_0 = 1 - \sin\phi \;\text{ or }\; K = \frac{\nu}{1-\nu} \tag{2}$$

where ϕ is the friction angle of soil and ν is Poisson's ratio.

Richards *et al.*[9] developed the concept of seismic fluidization to explain the stress path the free field must follow under seismic load if the granular soil is considered as an elastic, perfectly-plastic

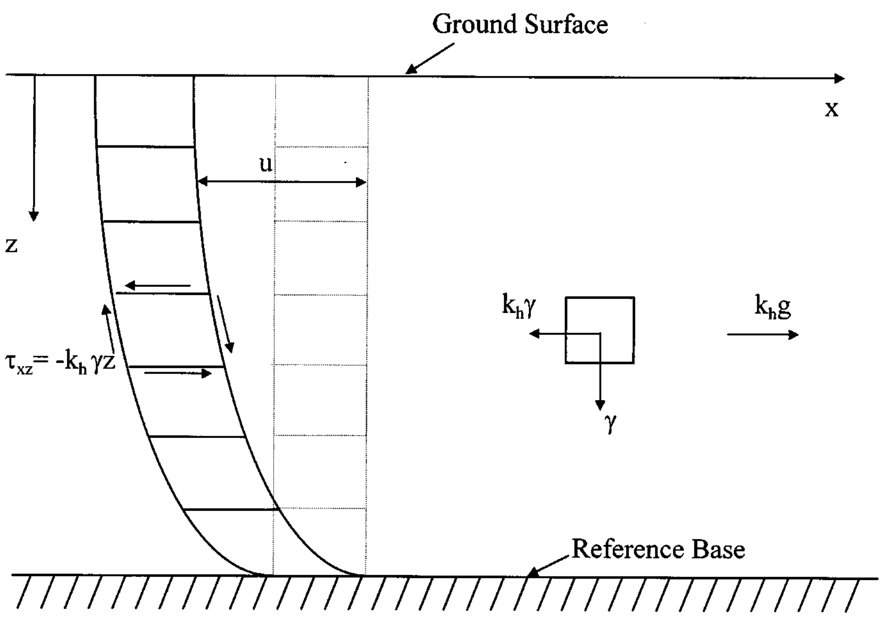


Figure 1. Inertial free-field with horizontal acceleration

material with a Mohr–Coulomb yield (failure) criterion. As the horizontal acceleration increases, the shear stress in the soil increases with a constant lateral earth pressure coefficient $K = K_0$ until the Mohr Circle of stress reaches the Mohr–Coulomb yield (failure) line. This state is called initial fluidization, or initial yield. The corresponding acceleration is

$$k_h^i = \frac{1}{2}\sqrt{(K_0 + 1)^2 \sin^2\phi - (K_0 - 1)^2} \tag{3}$$

The potential slip surfaces in the free field at this condition are inclined with respect to the horizontal direction, but due to the infinite lateral boundary, there is no failure surface possible at

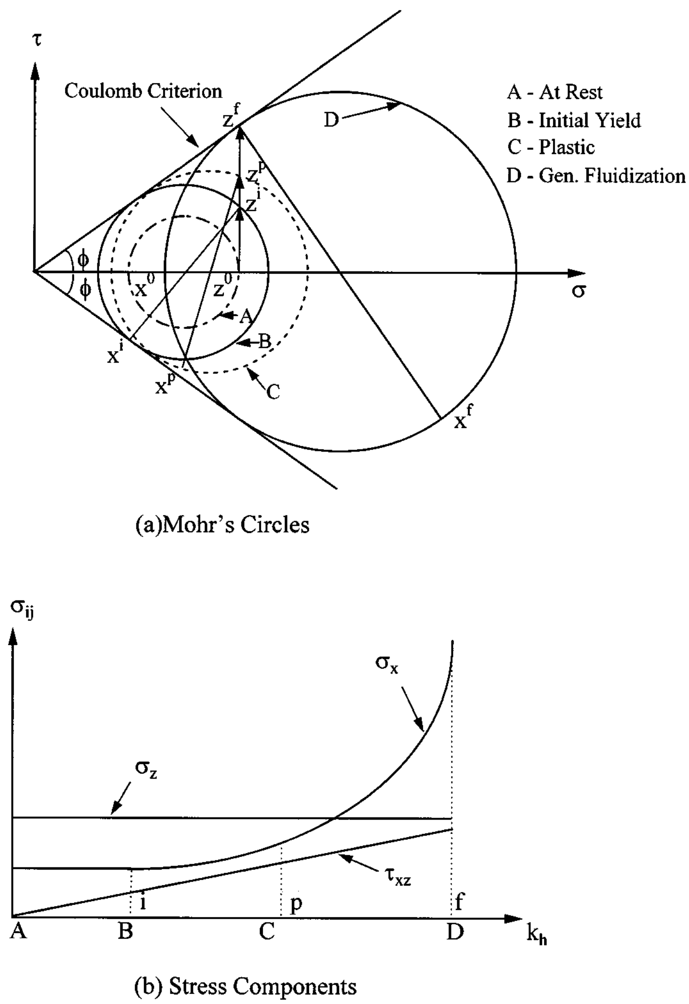


Figure 2. Stress states with increasing acceleration

this yield condition and the shear stress in the soil continues to increase as the acceleration increases. Thus, the value of the lateral earth pressure coefficient, K, must also increase with increasing horizontal acceleration to maintain equilibrium and to satisfy the Mohr–Coulomb yield criterion. The result for the plastic coefficient of lateral earth pressure derived by Richards *et al.*[9] is

$$K = K_{\mathrm{E}} = \frac{1 + \sin^2 \phi}{\cos^2 \phi} - \frac{2}{\cos^2 \phi} \sqrt{\sin^2 \phi - k_{\mathrm{h}}^2 \cos^2 \phi} \tag{4}$$

The stress field (equations (1)) continues to develop beyond yield until a state of general fluidization is reached where the yield surface throughout the whole layer is horizontal and a failure surface is developed at the base of the soil layer. The corresponding maximum acceleration $k_{\mathrm{h}}^{\mathrm{f}}$ is

$$k_{\mathrm{h}}^{\mathrm{f}} = \tan \phi \tag{5}$$

giving a maximum lateral earth pressure coefficient at failure

$$K = K_{\mathrm{EF}} = \frac{1 + \sin^2 \phi}{\cos^2 \phi} \tag{6}$$

Thus, equations (1) give the stress field throughout where equation (2) defines K in the elastic range, equation (4) defines K in the plastic range and equation (3) defines the transition or yield acceleration. The only parameter involved for a granular soil is the friction angle of the soil. The evolution of the Mohr's stress circles are shown in Figure 2(a) and the corresponding stress field is plotted in Figure 2(b).

Although the stress field solution derived here is based on a uniform horizontal acceleration with depth, the similar stress field can be derived in a similar way for any distribution of k_{h} as a function of z with or without vertical acceleration. In future work, such refinements or, for that matter, $k_{\mathrm{h}}(t)$ could be used to capture wave characteristics. However, the simplest assumption of a uniform k_{h} and $k_{\mathrm{v}} = 0$ are made so as to not obscure the basic derivation of the plasticity solution for strains and displacements.

INCREMENTAL THEORY OF PLASTICITY

In the plastic state, the total strain increments can be decomposed into elastic and plastic components as

$$\mathrm{d}\varepsilon_{ij} = \mathrm{d}\varepsilon_{ij}^{\mathrm{e}} + \mathrm{d}\varepsilon_{ij}^{\mathrm{p}} \tag{7}$$

The elastic strain increment, $\mathrm{d}\varepsilon_{ij}^{\mathrm{e}}$, can be determined directly from the generalized Hook's law in incremental form. The plastic strain increment, $\mathrm{d}\varepsilon_{ij}^{\mathrm{e}}$, can be determined by the plastic flow rule:

$$\mathrm{d}\varepsilon_{ij}^{\mathrm{p}} = \begin{cases} \mathrm{d}\lambda \dfrac{\partial Q}{\partial \sigma_{ij}}, & F = 0 \\ 0, & F < 0 \end{cases} \tag{8}$$

where F is the yield function, Q is a potential function and $\mathrm{d}\lambda$ is a positive proportional scaler. $Q = F$ is called the associative flow rule; and if $Q \neq F$, it is called nonassociative flow rule.

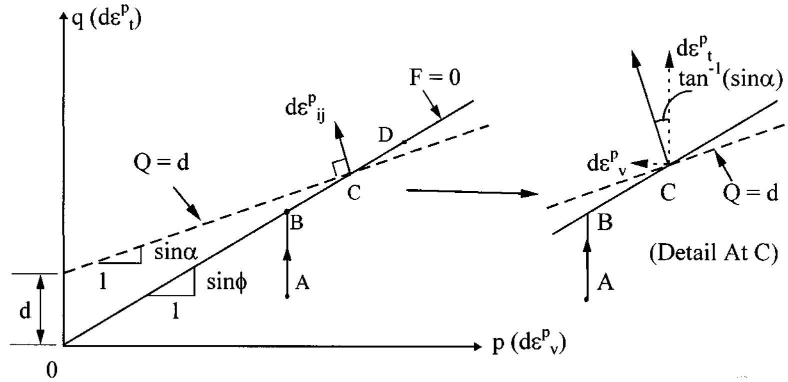


Figure 3. Non-associative flow rule

In the free field stress solution, the Mohr–Coulomb yield criterion of soil was used, where the intermediate principal stress σ_y (Figure 1) is not involved, the yield function can be expressed as

$$F = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} - \frac{1}{2}(\sigma_x + \sigma_z)\sin\phi = q - p\sin\phi \tag{9}$$

where ϕ is the friction angle of soil and the first two stress invariants are defined as $p = (\sigma_x + \sigma_z)/2$ and $q = \sqrt{(\sigma_x - \sigma_z)^2/4 + \tau_{xz}^2}$.

The potential function Q for the non-associative flow rule can be expressed as

$$Q = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} - \frac{1}{2}(\sigma_x + \sigma_z)\sin\alpha = q - p\sin\alpha \tag{10}$$

where α is a plastic parameter which may vary with the stress path. In the specific case when $\alpha = \phi$, the potential function Q is identical to the yield function F and the non-associative flow rule becomes the associative flow rule. For soil, α usually is less than ϕ. The stress path, yield function and the direction of plastic strain increment are shown in Figure 3.

PLASTIC DEFORMATION SOLUTION

For the free field, the stress, strain and displacements are a function of z only. Therefore,

$$\varepsilon_x = \varepsilon_y = 0 \quad \text{and} \quad \mathrm{d}\varepsilon_x = \mathrm{d}\varepsilon_y = 0 \tag{11}$$

From the condition $\mathrm{d}\varepsilon_y = 0$,

$$\sigma_y = [(1 - \nu)\, K_0 + \nu K_{\mathrm{E}}]\gamma z \tag{12}$$

and from $\mathrm{d}\varepsilon_x = 0$, it can be shown that

$$\mathrm{d}\lambda = -\frac{\mathrm{d}\varepsilon_x^{\mathrm{e}}}{\partial Q/\partial\sigma_x} \tag{13}$$

In order to satisfy the Drucker's postulate[11] of positive plastic work:

$$\begin{aligned} \mathrm{d}w_{\mathrm{I}}^{\mathrm{P}} &= \sigma_{ij}\mathrm{d}\varepsilon_{ij}^{\mathrm{P}} \geqslant 0 \\ \mathrm{d}w_{\mathrm{II}}^{\mathrm{P}} &= \mathrm{d}\sigma_{ij}\mathrm{d}\varepsilon_{ij}^{\mathrm{P}} \geqslant 0 \end{aligned} \tag{14}$$

the plastic parameter α in the potential function must satisfy the condition:

$$\sin\alpha \geqslant \frac{\sigma_x - \sigma_z}{\sqrt{(\sigma_x - \sigma_z)^2 + 4\tau_{xz}^2}} \tag{15}$$

which, assuming $\sin\alpha = a\sin\phi$, can be expressed as

$$a \geqslant a_0 = \frac{\sigma_x - \sigma_z}{\sin\phi\sqrt{(\sigma_x - \sigma_z)^2 + 4\tau_{xz}^2}} \tag{16}$$

For the associative flow rule where $\alpha = \phi$, $a = 1$ and the dilation of soil may be overestimated.[12] Expressing stresses in terms of the coefficient of horizontal acceleration k_{h}, the possible range of **a** is the shaded area in Figure 4. At the limit condition when $a = a_0$ in equation (16), the scalar $\mathrm{d}\lambda$ becomes infinite and so do the plastic strains.

Because $\mathrm{d}\varepsilon_y^{\mathrm{p}}$ is zero, the plastic volumetric strain is expressed as

$$\mathrm{d}\varepsilon_{\mathrm{v}}^{\mathrm{p}} = \mathrm{d}\varepsilon_x^{\mathrm{p}} + \mathrm{d}\varepsilon_z^{\mathrm{p}} = -\sin\alpha\,\mathrm{d}\lambda \tag{17}$$

when $\alpha > 0$, the plastic volumetric strain is always negative (expansion) and when $\alpha < 0$, the plastic volumetric strain is positive (contraction). From the definition of a, the plastic volumetric strain increment, $\mathrm{d}\varepsilon_{\mathrm{v}}^{\mathrm{p}}$, and coefficient, a, always have different signs. When $a > 0$, $\mathrm{d}\varepsilon_{\mathrm{v}}^{\mathrm{p}} < 0$ or dilative; when $a < 0$, $\mathrm{d}\varepsilon_{\mathrm{v}}^{\mathrm{p}} > 0$ or the soil is compressed; when $a = 0$, there is no plastic volumetric strain. To calculate the plastic strains in the plastic range from initial yield ($k_{\mathrm{h}} = k_{\mathrm{h}}^{\mathrm{i}}$) to general fluidization ($k_{\mathrm{h}} = k_{\mathrm{h}}^{\mathrm{i}}$), either $a = 1$ for the associative flow rule or the coefficient, a, must vary. In terms of horizontal acceleration, the parameter **a** can be expressed as

$$a = \frac{\sin\alpha}{\sin\phi} = (1 - b)\left(\frac{k_{\mathrm{h}} - k_{\mathrm{h}}^{\mathrm{i}}}{k_{\mathrm{h}}^{\mathrm{f}} - k_{\mathrm{h}}^{\mathrm{i}}}\right)^r + b, \quad (b \leqslant 1) \tag{18}$$

where r and b are two constant material parameters which can be chosen according to the properties of soil to insure that equation (16) is satisfied. For the special case of $\alpha = \phi$, $b = 1$ and when $b = 0$ and $r = \infty$, $\alpha = 0$.

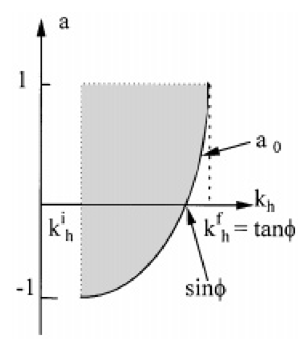


Figure 4. Possible values of parameter a

Combining equation (10) and equation (18) a specific potential function is defined. The plastic strain can be determined by using the flow rule equation (8) with this potential function, the proportional scaler in equation (13) and the free-field stress solution.

The total strain increments of soil in the free field after initial yield are, therefore,

$$
\begin{aligned}
\mathrm{d}\gamma_{xz} &= \mathrm{d}\gamma_{xz}^{\mathrm{e}} + \mathrm{d}\gamma_{xz}^{\mathrm{p}} \\
&= -\frac{\gamma z\,\mathrm{d}k_{\mathrm{h}}}{G} - \frac{16(1-v)\,k_h}{(\sin\alpha\sqrt{(K_{\mathrm{E}}-1)^2+4k_{\mathrm{h}}^2}-K_{\mathrm{E}}+1)\sqrt{\sin^2\phi-k_{\mathrm{h}}^2\cos^2\phi}}\,\frac{\gamma z \mathrm{d}k_{\mathrm{h}}}{G} \\
\mathrm{d}\varepsilon_z &= \mathrm{d}\varepsilon_z^{\mathrm{e}} + \mathrm{d}\varepsilon_z^{\mathrm{p}} \\
&= -\frac{8v\gamma z\,\mathrm{d}k_{\mathrm{h}}}{G\sqrt{\sin^2\phi-k_{\mathrm{h}}^2\cos^2\phi}} - \frac{4(1-v)\sin\alpha\sqrt{(K_{\mathrm{E}}-1)^2+4k_{\mathrm{h}}^2}+K_{\mathrm{E}}-1)\,\gamma z\,\mathrm{d}k_{\mathrm{h}}}{(\sin\alpha\sqrt{(K_{\mathrm{E}}-1)^2+4k_{\mathrm{h}}^2}-K_{\mathrm{E}}+1)\,G\sqrt{\sin^2\phi-k_{\mathrm{h}}^2\cos^2\phi}}
\end{aligned}
\tag{19}
$$

where G is the initial elastic shear modulus, K_{E}, the seismic coefficient of lateral earth pressure as given by equation (4), and $\sin\alpha$ as shown in equation (18) are functions of k_{h}.

For soil is in the elastic state, $k_{\mathrm{h}} < k_{\mathrm{h}}^{\mathrm{i}}$, the strains are:[9]

$$
\gamma_{xz} = -\frac{\gamma z k_{\mathrm{h}}}{G}, \quad \varepsilon_z = \varepsilon_x = \varepsilon_y = 0 \tag{20}
$$

The total elastoplastic strain is the sum of the elastic strain before initial yield (equation (20)) and the elastoplastic strain beyond the initial yield obtained by integrating equation (19) from initial yield acceleration, $k_{\mathrm{h}}^{\mathrm{i}}$, to the applied acceleration k_{h}. The total elastoplastic strain can then expressed as

$$
\begin{aligned}
\gamma_{xz} &= f_1(k_{\mathrm{h}})\,\frac{z}{G} \\
\varepsilon_z &= f_2(k_{\mathrm{h}})\,\frac{z}{G}, \quad \varepsilon_x = \varepsilon_y = 0
\end{aligned}
\tag{21}
$$

where $f_1(k_{\mathrm{h}})$ and $f_2(k_{\mathrm{h}})$ are two functions of the horizontal acceleration. In the elastic state, $f_1(k_{\mathrm{h}}) = -\gamma k_{\mathrm{h}}$ and $f_2(k_{\mathrm{h}}) = 0$. However in the plastic state, finding an explicit expression for $f_1(k_{\mathrm{h}})$ and $f_1(k_{\mathrm{h}})$ may not be possible when a varies. Nevertheless, since in equation (19) the elastic shear modulus G is only a function of the depth, only the magnitude of the plastic strains depend on the acceleration level and their distribution does not.

The displacement can be determined by integrating the strains

$$
u = -\int \gamma_{xz}\,\mathrm{d}z, \qquad w = -\int \varepsilon_z\,\mathrm{d}z, \quad v = 0 \tag{22}
$$

using the given displacement boundary condition that $u = w = 0$ at $z = H$. If the elastic shear modulus is expressed as

$$
G = G_1\left(\frac{z}{H}\right)^n \tag{23}
$$

where n is the parameter for different distribution and G_1 is the shear modulus at depth H then the displacements may be expressed as

$$
\begin{aligned}
u &= f_1(k_{\mathrm{h}})\,\frac{H^2 - H^n z^{2-n}}{G_1(2-n)} \\
w &= f_2(k_{\mathrm{h}})\,\frac{H^2 - H^n z^{2-n}}{G_1(2-n)}
\end{aligned}
\tag{24}
$$

For granular soil, n is usually no larger than 1. As seen in equation (24), the shape of the elastoplastic horizontal displacement using either the associative or non-associative flow rule is the same as that from the elastic solution. In general, the displacements can only be solved by numerical methods and the closed-form solutions are only available when $f_1(k_{\mathrm{h}})$ and $f_2(k_{\mathrm{h}})$ can be expressed explicitly.

CLOSED-FORM SOLUTION FOR TWO SPECIAL CASES

In some special cases, the closed-form solution for elastoplastic strains in equation (21) can be derived and the closed-form solution for the displacement field can be found. For example, in the case where $\alpha = \phi$ (associative flow rule) and the soil expands at any acceleration. The elastoplastic strain field is expressed as

$$
\begin{aligned}
\gamma_{xz} &= -\frac{\gamma z k_{\mathrm{h}}}{G} + \frac{2(1-v)\gamma z}{G\cos^2\phi}\left[k_{\mathrm{h}} - k_{\mathrm{h}}^{\mathrm{i}} - \frac{1}{2}\tan\phi \ln\frac{A(k_{\mathrm{h}})}{A(k_{\mathrm{h}}^{\mathrm{i}})}\right] \\
\varepsilon_z &= -\frac{v\gamma z}{2G}(K_{\mathrm{E}} - K_0) + \frac{(1-v)\gamma z}{2G\cos^2\phi}\left[4\tan^2\phi \ln\frac{B(K_{\mathrm{E}})}{B(K_0)} + (1+\sin^2\phi)\,(K_{\mathrm{E}} - K_0)\right]
\end{aligned}
\tag{25}
$$

where

$$
\begin{aligned}
A(k_{\mathrm{h}}) &= \frac{\sin\phi + k_{\mathrm{h}}\cos\phi}{\sin\phi - k_{\mathrm{h}}\cos\phi}, \qquad A(k_{\mathrm{h}}^{\mathrm{i}}) = \frac{\sin\phi + k_{\mathrm{h}}^{\mathrm{i}}\cos\phi}{\sin\phi - k_{\mathrm{h}}^{\mathrm{i}}\cos\phi} \\
B(K_{\mathrm{E}}) &= (K_{\mathrm{E}}+1)\cos^2\phi - 2, \qquad B(K_0) = (K_0+1)\cos^2\phi - 2
\end{aligned}
\tag{26}
$$

If the distribution of shear modulus of soil is expressed by equation (23), the displacements of the soil are:

$$
\begin{aligned}
u &= -\frac{\gamma k_{\mathrm{h}}(H^2 - H^n z^{2-n})}{(2-n)\;G_1} + \frac{2(1-v)\gamma(H^2 - H^2 z^{2-n})}{(2-n)\;G_1\cos^2\phi}\left[k_{\mathrm{h}} - k_{\mathrm{h}}^{\mathrm{i}} - \frac{1}{2}\tan\phi\ln\frac{A(k_{\mathrm{h}})}{A(k_{\mathrm{h}}^{\mathrm{i}})}\right] \\
w &= -\frac{v\gamma(H^2 - H^n z^{2-n})}{2(2-n)\;G_1}(K_{\mathrm{E}} - K_0) + \frac{(1-v)\gamma(H^2 - H^n z^{2-n})}{2(2-n)\;G_1\cos^2\phi} \\
&\quad\times\left[4\tan^2\phi\ln\frac{B(K_{\mathrm{E}})}{B(K_0)} + (1+\sin^2\phi)\,(K_{\mathrm{E}} - K_0)\right]
\end{aligned}
\tag{27}
$$

A closed-form solution can be derived for the case $\alpha = 0$ where the soil is at the critical void ratio and there is no plastic volumetric strain. The elastoplastic strains are:

$$\gamma_{xz} = -\frac{\gamma z k_h}{G} + \frac{2(1-\nu)\gamma z}{G} I(k_h)$$
$$\varepsilon_z = -\frac{\nu \gamma z}{2G}(K_E - K_0) + \frac{(1-\nu)\gamma Z}{2G}(K_E - K_0) \tag{28}$$

where

$$I(k_h) = (k_h - k_h^i) - \tan\phi \sin\phi \left(\sin^{-1}\sqrt{1 - k_h^2 \cot^2\phi} - \sin^{-1}\sqrt{1 - k_h^{i2} \cot^2\phi} \right) + \sin\phi \ln \frac{D(k_h)}{D(k_h^i)}$$

$$D(k_h) = \frac{[k_h \cos^2\phi + 1 - C(k_h)]\sin\phi}{C(k_h) - \sin^2\phi}; \quad D(k_h^i) = \frac{[k_h^i \cos^2\phi + 1 - C(k_h^i)\ ]\sin\phi}{C(k_h^i) - \sin^2\phi} \tag{29}$$

$$C(k_h) = \sqrt{\sin^2\phi - k_h^2 \cos^2\phi}; \quad C(k_h^i) = \sqrt{\sin^2\phi - k_h^{i2} \cos^2\phi}$$

If the distribution of shear modulus is also expressed by equation (23), the displacements are:

$$u = -\frac{\gamma k_h (H^2 - H^n z^{2-n})}{(2-n)G_1} + \frac{2(1-\nu)\gamma(H^2 - H^n z^{2-n}) I(K_h)}{(2-n)G_1}$$
$$w = -\frac{\nu\gamma(H^2 - H^n z^{2-n})}{2(2-n)G_1}(K_E - K_0) + \frac{(1-\nu)\gamma z(H^2 - H^n z^{2-n})}{2(2-n)G_1}(K_E - K_0) \tag{30}$$

However as shown in Figure 4, the non-associative flow rule $\alpha = 0$ (or $a = 0$) can only be applicable up to $k_h = \sin\phi$ where $\sigma_x = \sigma_z$ at which point the shear strain and horizontal displacement become infinite. This is understandable. In the plastic state the horizontal stress σ_x increases as the horizontal acceleration increases while the vertical stress σ_z remains constant. If σ_x becomes larger than the vertical stress σ_z, the zero strain condition in the y direction forces the soil to expand in the vertical direction and the assumption of zero plastic volumetric strain can no longer be valid. The observation that even 'loose' granular material shows dilation near failure[13] may reflect this analytic result. For $\alpha = 0$, the vertical displacement is always a settlement with a limited value.

EFFECTS OF DIFFERENT PLASTIC PARAMETERS

The soil deformation, in general, may not be calculated by applying the plastic flow rule with a constant value of α or a. These parameters will vary at different load levels. To study the effects of parameters r and b in equation (18) which define these variation, it is assumed that the depth of soil $z = 0{\cdot}406$ m (16 in), $\phi = 38^\circ$, $\gamma = 1763$ kg/m^3 (110 1b/ft^3) and the elastic shear modulus $G = 12{\cdot}983$ MPa (1883 psi). As a first case, r is fixed at 2 and b is varied. The related parameter a is plotted in Figure 5(a). The shear strain versus normalized shear stress is plotted in Figure 5(b). The volumetric strain versus shear strain is plotted in Figure 5(c). It can be seen that as b decreases the shear strain increases, and the dilation decreases.

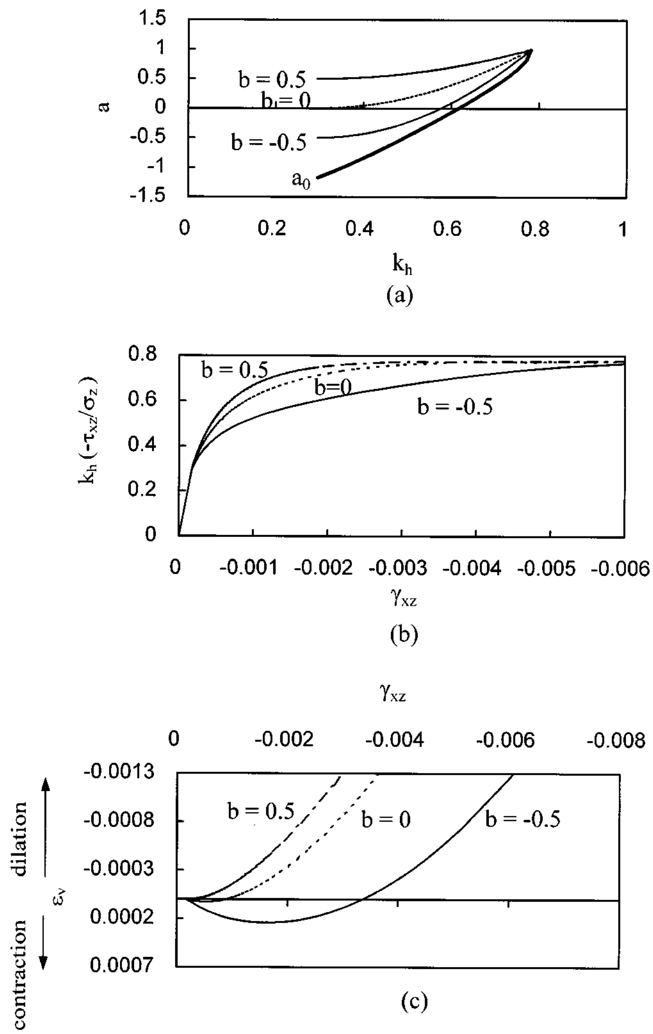
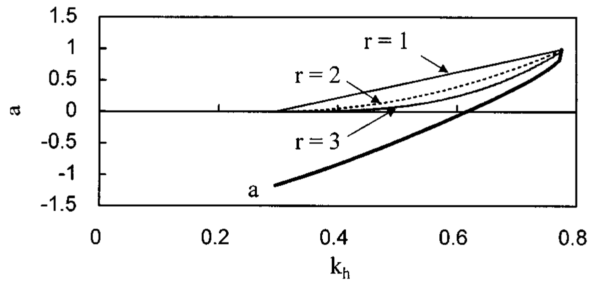


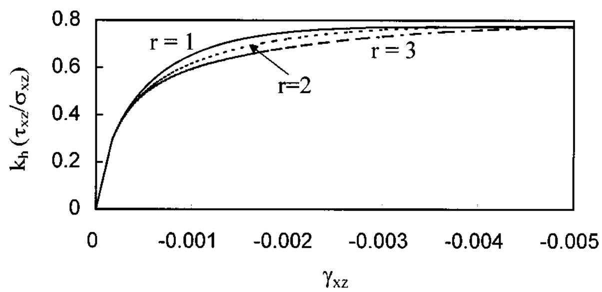
Figure 5. Effects due to plastic parameter $r = 2$ and different b; (a) coefficient a; (b) shear strain and stress; (c) volumetric strain and shear strain

As a second case, b is fixed ($b = 0$) and r is varied. As r increases, the difference between a and a_0, or $a - a_0$, decreases (Figure 6(a)), the shear strain increases (Figure 6(b)), the volumetric strain becomes less negative (dilation decreases) with respect to the shear strain level (Figure 6(c)).

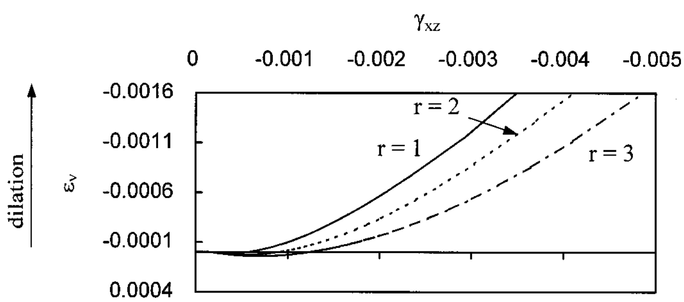
The effects of parameters r and b can also be deduced from their contributions to the values of a in equation (18). As r is increased or b is decreased, the difference between a and a_0 (as shown in Figures 5(a) and 6(a)) is decreased and the rate of the strain increment is increased. When the value of a is negative, the soil is in volumetric compression. When a is positive, the soil is dilating.



(a)



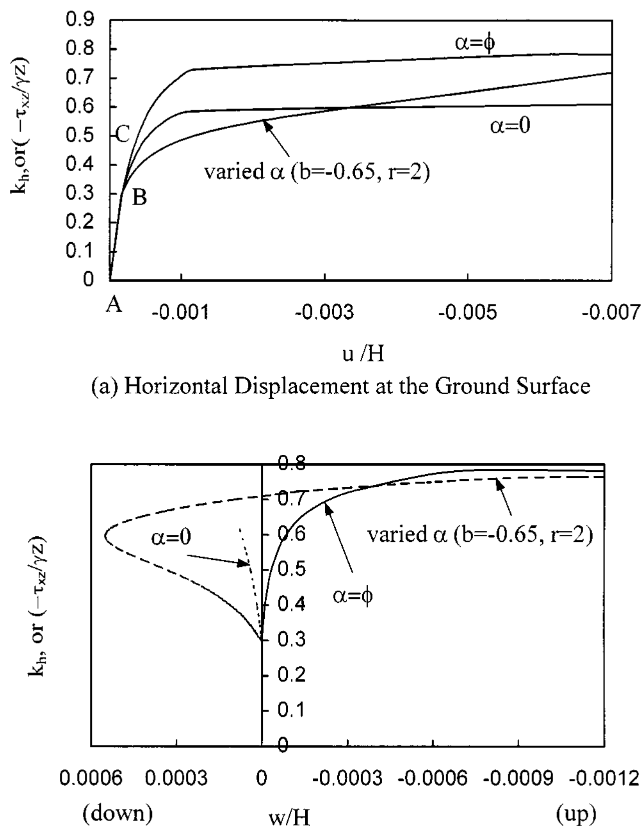
(b)



(c)

Figure 6. Effects due to plastic parameter $b = 0$ and different r: (a) coefficient a; (b) shear strain and stress; (c) volumetric strain and shear strain

For a known distribution of shear modulus, the displacement of soil can be determined by integration of soil strains. Figure 7 shows the soil displacement at the ground surface for the case where $b = -0{\cdot}65$, $r = 2$ compared to the displacement from the two special cases $\alpha = \phi$ (or $r = 0$) and $\alpha = 0$ (or $b = 0$ & $r \rightarrow \infty$). Here $H = 0{\cdot}91$ m(36 in), $\gamma = 1763$ kg/m^3(110 lb/ft^3), $\phi = 38^\circ$, $G = G_1\sqrt{z/H}$ and $G_1 = 19{\cdot}596$ MPa (2842 psi). With varied α the horizontal displacement is larger than those from the other two special cases at a medium acceleration and increases gradually with a value between those from the other two cases at large accelerations. The vertical displacement for a flow rule with varied α is settlement at first but after yield the top surface



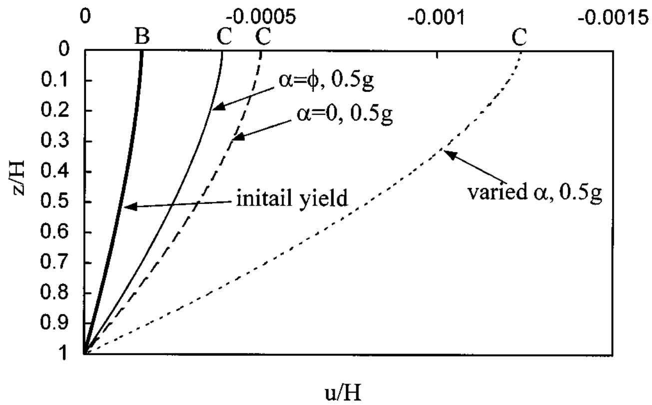
(a) Horizontal Displacement at the Ground Surface

(b) Vertical Displacement at the Ground Surface

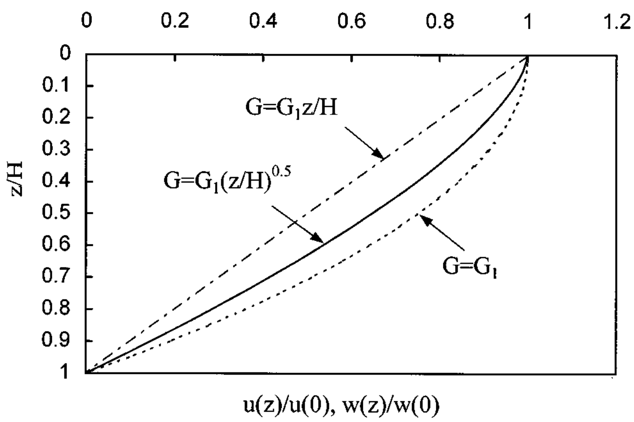
Figure 7. Displacements at the ground surface from three different plastic flow rules ($\phi = 38°$)

moves up. The vertical displacement for the flow rule with $\alpha = \phi$ always moves up and for $\alpha = 0$ always moves down. Therefore, the displacement from a flow rule with varied a is probably more reasonable than either special case.

The distribution of horizontal displacement of soil for these three different cases at the acceleration for initial yield and at $0{\cdot}5g$ are plotted in Figure 8(a). Because the shape of the displacement distribution is the same based on the distribution of the initial elastic shear modulus of soil, the normalized displacement distribution of these curves are the same (Figure 8(b)). The normalized horizontal displacements of soil with a uniform and linear distribution of $G(z)$ ($n = 0, 1$ in equation (23)) are also plotted in the Figure 8(b). As shown in equation (24), the distribution of vertical displacement is similar to that of the horizontal displacement. Thus, the distribution of the normalized vertical displacement is exactly the same as that of the normalized horizontal displacement as shown in Figure 8(b).



(a)



(b)

Figure 8. (a) Distribution of horizontal displacement at initial yield and 0·5g, $G = G_1(z/H)^{0.5}$ and (b) normalized distribution of horizontal and vertical displacements

DEFORMATION FROM A SIMPLE NON-LINEAR ANALYSIS

Based on the observed response from shaking table testing, a constitutive model was proposed by Fishman *et al.*[14] to describe the free-field behaviour of dry sand during seismic loading. The shear stress–strain relation can be expressed as

$$\tau_{xz} = G_0\gamma_{xz}\left(Q + \frac{1-Q}{\left(1+\left|\dfrac{G_0\gamma_{xz}}{\gamma z\tan\phi}\right|^R\right)^{1/R}}\right) \tag{31}$$

Where τ_{xz} and γ_{xz} are shear stress and strain, z is the depth of the stress point, G_0 is initial shear modulus of soil, γ is the unit weight of the sand, ϕ is the fraction angle of the sand, Q is the ratio of

the post-yield shear modulus to the initial shear modulus, R is a parameter related to the friction angle to the soil given as

$$R(\phi) = \frac{\ln 2}{\ln\left(\dfrac{1 - Q}{(k_h^i/\tan\phi) - Q}\right)} \tag{32}$$

where k_h^i is the acceleration coefficient at initial yield of the soil.

For a given shear stress level the shear strain may be determined by dividing the stress by the equivalent shear modulus. The equivalent shear modulus predicted with equation (31) was very close to those measured in the shaking table test according to Fishman *et al.*.[14] This constitutive model may also be used to calculate the horizontal displacement by integrating shear strain determined from equation (31) by a numerical integration.

COMPARISON BETWEEN THE ANALYSIS AND THE SHAKING TABLE TESTING

Test measurements from the shaking table test can be compared to the calculated results. The seismic response of the soil in the free field is simulated by testing in a seismic test chamber filled with dry sand on the shaking table at the National Centre for Earthquake Engineering Research (NCEER) at Buffalo. The test chamber is 4·27 m (14 ft) long, 0·91 m (3 ft) wide and 1·22 m (4 ft) high. Ottawa sand (ASTM C-109) was used in the test box in the air dry condition. Pluviation was used to place the sand in the test chamber. The depth of the sand layer was 0·91 m (3 ft). Flexible end walls were installed vertically at the ends. Accelerometers were placed at four different depths within the sand layer and displacement transducers were placed on the surface of the sand. Details of the test chamber and associated testing are given by Fishman *et al.*.[14] The test box was placed on the shaking table and the horizontal acceleration was applied.

As described by Fishman *et al.*,[14] the friction angle of sand in the test box was 38° and the Poisson's ratio was 0·28. The density of sand was about 1763 kg/m^3 (110 pcf). The average elastic shear modulus of soil was 14·6 MPa (2117 psi). The displacements from the tests are the peak displacements measured under a single horizontal 5 Hz sinusoidal acceleration pulse at different amplitudes.

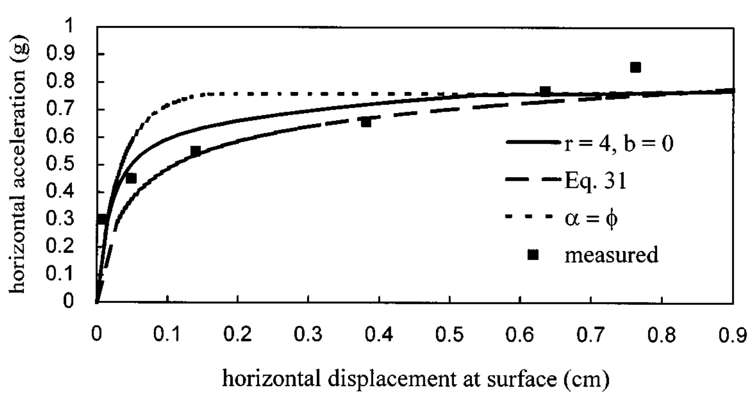


Figure 9. Calculated and measured soil surface horizontal displacement

In the analysis the distribution of elastic shear modulus of soil is assumed as that in equation (23) with $n = 0{\cdot}5$ and $G_1 = 19{\cdot}5$ MPa (2823 psi). In the plastic analysis, the associative flow rule and the non-associative flow rule with $r = 4$ and $b = 0$ are used. In the simple analysis method (equation (31)) the parameters of $Q = 0{\cdot}0075$ and $R = 0{\cdot}7$ are used.

The analytical horizontal displacement at the surface of soil and those measured from the shaking table tests are plotted in Figure 9. It can be seen that horizontal displacements calculated by the plastic analysis method with the non-associative flow rule ($r = 4$ and $b = 0$) are close to those measured in the tests and those calculated by the simple method in equation (31).

CONCLUSION

From the elastoplastic seismic free-field stress solution, a corresponding deformation analysis for dry granular soil is developed using the theory of plastic flow. The earthquake loading is analysed based on the pseudostatic method, and the soil is assumed as an elastic, perfectly plastic material with Mohr–Coulomb yield criterion. The assumption of uniform horizontal acceleration, which is practical and commonly used for a shallow layer of soil near the ground surface, is used in the analysis.

In the analysis, a special function (equation (18)) with two material constants, b and r, to represent the plastic parameter α (or **a**) in the potential function (equation (10)) is developed. The effects due to different material constants are investigated. In general, the deformation of soil can be found by numerical methods although closed-form solutions are derived for two special cases, $\alpha = \phi$ and $\alpha = 0$. It is found that in the case of the associative flow rule ($\alpha = \phi$), the dilation of soil is usually overpredicted. For the case of the non-associative flow rule with the assumption of no plastic volumetric strain ($\alpha = 0$), the solution is only applicable for a part of the total acceleration range determined in the stress solution. The more reasonable deformation for the whole acceleration range can be determined by using a non-associative flow with varied value of α. It is suggested that the material constants for dense sand may be roughly $b = 0$ and $r = 4$ since calculated surface horizontal displacement of soil compares well with those measured from a shaking table test.

In the soil model developed here, the yield of soil and the degradation of shear modulus depend on the shear stress, or the horizontal acceleration. With the assumption of uniform horizontal acceleration in the soil layer, the shapes of the elastic displacement and elastoplastic displacements, even with different flow rules, are identical. Hence the elastoplastic horizontal displacement solution may be expressed in the form of the elastic solution but with different values of an equivalent shear modulus instead of the elastic shear modulus. The shape of the horizontal displacement distribution with depth is very important for the study of soil–structure interaction. In some cases the deformation shape may be more important than the magnitude of the displacements. This was shown in the analysis of retaining structures.[10] The conclusion of the identical deformation shapes for both elastic and plastic deformations is generally not valid if non-uniform horizontal acceleration is applied.

The focus of the free-field analysis here is to determine the maximum values of stresses, strains and displacements under inertia loading beyond yield. This is justified on the basis that in an earthquake there are only a few peaks of acceleration higher than the acceleration for initial yield and the effects due to these few extreme cycles on the calculation of maximum displacements are most important for engineering design. Such examples include retaining structure design and the

seismic bearing capacity of footings both of which are related to the plastic horizontal displacement and maximum stress.

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